Online Appendix for

Decomposing the difference between multidimensional well-being inequality and income inequality: method and application

Marko Ledić and Ivica Rubil

Published in

Research on Economic Inequality vol.27, K. Decancq and P. Van Kerm (eds),

Emerald Publishing, 2019.

In this online appendix, we discuss how the marginal contribution of each dimension is determined. Consider an example with two dimensions, labelled A and B and one person. There are two ways in which dimension A can contribute to her total WTP, and two corresponding marginal contributions of A. The first marginal contribution is $WTP(\mathrm{A}) - WTP(\emptyset)$, where $WTP(\mathrm{A})$ is the willingnes-to-pay when the person is potentially deprived in A only, and $WTP(\emptyset) \equiv 0$ is the willingness-to-pay when there are no potential deprivations.[1] The second is $WTP(\mathrm{A}, \mathrm{B}) - WTP(\mathrm{B})$, where $WTP(\mathrm{A}, \mathrm{B}) \equiv WTP$ is the willingness-to-pay when the person is potentially deprived in both dimensions, and $WTP(\mathrm{B})$ is the willingness-to-pay when the person is potentially deprived in dimension B only. These two marginal contributions will generally differ, and the issue is whether to use one or the other when measuring the contribution of A to the total willingness-to-pay.

A common practice in such situations is to use the so-called Shapley decomposition.[2] By this approach, the contribution of dimension d to the total willingness-to-pay is obtained as the weighted sum of all possible marginal contributions of that dimension. In our example, the contribution of A is obtained as the equally weighted sum of the two marginal contributions: [3]

$$WTP_{\mathrm{A}} = (1/2) \cdot \big(WTP(\mathrm{A}) - WTP(\emptyset)\big) + (1/2) \cdot \big(WTP(\mathrm{A}, \mathrm{B}) - WTP(\mathrm{B})\big).$$

The logic behind equal weights in this example is the following. There are two ‘types’ of marginal contributions: ‘type 1’, obtained considering only the case where the person is potentially deprived in A only (one potential deprivation); and ‘type 2’, obtained considering only the case where the person is potentially deprived in A and B (two potential deprivations). By the Shapley approach, the weights are equal across types, and if there is more than one marginal contribution of a particular type, the weight assigned

[1] The magnitude of deprivation matters as well. For a given person, $WTP(\mathrm{A}) - WTP(\emptyset)$ increases with the shortfall of her achievement in A from the perfect level. But here we focus on one particular ‘realisation’ of deprivation in A, namely the actual deprivation. Also, for two persons with identical preferences, $WTP(\mathrm{A}) - WTP(\emptyset)$ is larger for the one whose achievement in A is farther from the perfect achievement (i.e., the one who is more deprived in A).

[2] On the use of the Shapley decompositions in distributional analysis, see Shorrocks (2013). See Hierro, Gomez-Alvarez and Atienza (2012) for an application in the taxation context.

[3] One must be careful to distinguish between WTP_d and $WTP(d)$, d = A, B.

to that type is split equally among these marginal contributions. In our example, there are two types and one marginal contribution per type, implying equal weights of 1/2.[4] WTP_{B} is obtained analogously.

With WTP_{A} and WTP_{B} at hand, we can do the detailed decompositions $V = V_{\mathrm{A}} + V_{\mathrm{B}}$ and $R = R_{\mathrm{A}} + R_{\mathrm{B}}$. Take V_{A}. Again, there are two distinct marginal contributions of WTP_{A} to V_{A}: $C_{\mathrm{A}}^{*} - G$ and $(C_{\mathrm{A,B}}^{*} - G) - (C_{\mathrm{B}}^{*} - G) = (C_{\mathrm{A,B}}^{*} - C_{\mathrm{B}}^{*})$, where C_{d}^{*} is the concentration index of $Y_{d}^{*} = Y - WTP_{d}$ (d = A, B), and $C_{\mathrm{A,B}}^{*}$ the concentration index of $Y_{\mathrm{A,B}}^{*} = Y - WTP_{\mathrm{A}} - WTP_{\mathrm{B}}$. Following the described logic of the Shapley approach, the two marginal contributions are again equally weighted and added up to get V_{A}. By analogy, one can easily obtain V_{B}, as well as R_{A} and R_{B}.

In a general case with the total number of dimensions D, the weight of a type-M marginal contribution of dimension d is given by the formula

$$w_{d}(D, M) = \frac{(D-M)!(M-1)!}{D!} \; M \in \{1,2, \dots, D\}.$$

References

Hierro, L. A., Gómez-Alvarez, R., & Atienza, P. (2012). The contribution of US taxes and social transfers to income redistribution. *Public Finance Review*, 40, 381–400.

Shorrocks, A. (2013). Decomposition procedures for distributional analysis: A unified framework based on the Shapley value. *Journal of Economic Inequality*, 11, 99–126.

[4] By the same logic, with three non-income dimensions, there are three types of marginal contributions for a particular dimension—type 1, type 2, and type 3—and each is assigned the weight of 1/3. There is one type-1 contribution weighted by 1/3; two type-2 marginal contributions, each weighted by 1/6; and one type-3 marginal contribution weighted by 1/3.